Design tradeoffs for X-ray telescopes

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Contents

1 Introduction

We have investigated the design tradeoffs for X-ray telescopes which utilize the following geometries and technology implementations.

- Nested Wolter I - Silicon Pore Optics (SPO)
- Nested Wolter I - Micro Pore Optics (MPO)
- Lobster eye Schmidt geometry (Kirkpatrick-Baez variant) - float glass plates
- Lobster eye Schmidt geometry (Kirkpatrick-Baez variant) - SPO
- Lobster eye square pore geometry - MPO

The purpose of the tradeoff is to optimize the performance characteristics of the X-ray optics including angular resolution, collecting area vs. photon energy, field of view and off-axis response, both vignetting and degraded angular resolution. We consider inherent aberrations which arise from the geometry, the geometric limitations imposed by the technology and the manufacture and integration error budget of the technology which limits the scientific performance of the X-ray telescope optics.

The tradeoff has been performed using a combination of analytical optimization, ray tracing simulation and computer modelling using the QSOFT software package.

2 Wolter I

2.1 Wolter I - SPO

2.2 Wolter I - MPO

3 Lobster Eye

3.1 Analytic optimization

Within AHEAD project, analytic method of calculation and maximization of effective collecting length of lobster eye system was explored. The method can be used for

Schmidt as well as for Angel designs. In comparison to ray-tracing simulation, the analytical method is very quick and needs only few simple equations to be evaluated. The common disadvantage of analytical methods is that they are less exact. Usually, analytical methods are used to find the initial point for consequent optimization by ray-tracing methods. Moreover, analytical methods give understanding how the result depends on input parameters.

General analytic formula for calculating of effective collecting length of lobster eye (LE) was found in the form

$$L(r, a, t, \zeta) = 2r\frac{a}{a+t}K(\zeta), \tag{1}$$

where

$$K(\zeta) := \frac{\widetilde{\mathcal{R}}(2\zeta) - 2\widetilde{\mathcal{R}}(\zeta) + \tilde{R}(0)}{\zeta}. \tag{2}$$

Here, r is the LE system radius, a is middle mirror distance (or chamber size), t is mirror thickness (or chamber wall width). $\zeta = a/h$ is effective angle of the system, where h is mirror (chamber) depth. It is supposed that a reflectivity model $R(\beta)$ is known, here β is a grazing angle. $\widetilde{\mathcal{R}}(\beta) := \int\int \mathcal{R}(\beta)\mathrm{d}\beta^2$ is (arbitrary) second antiderivative of R.

The formula was evaluated for these reflectivity models:

- Single step model:

$$\mathcal{R}(\theta) = \begin{cases} Q & \forall 0 \le \theta < \kappa \\ 0 & \forall \theta > \kappa \end{cases} \tag{3}$$

- Linear model:

$$\mathcal{R}(\theta) = \begin{cases} Q\left(1 - \frac{\theta}{\kappa}\right) & \forall 0 \le \theta \le \kappa \\ 0 & \forall \theta \ge \kappa \end{cases} \tag{4}$$

- Smoothed step model:

$$\mathcal{R}(\theta) = \begin{cases} Q & \forall 0 \le \theta \le \rho \\ Q\frac{\kappa-\theta}{\kappa-\rho} & \forall \rho \le \theta \le \kappa \\ 0 & \forall \theta \ge \kappa \end{cases} \tag{5}$$

These models are commonly used for plenty of materials.

The effective angle ζ has be proved to be the key parameter to be optimised to maximize the effective collecting length. For named models, formulae for optimal value of ζ were found as

- Single step model: $\zeta_{\text{optimal}} = \frac{\sqrt{2}}{2}\kappa \doteq 0.707\kappa$

- Linear model: $\zeta_{\text{optimal}} = \frac{\kappa}{2}$
- Smoothed step model

$$\zeta_{\text{optimal}} = \begin{cases} \left\{\cos\left[\frac{1}{3}\arccos\left(\sigma^3\right) + \frac{\pi}{3}\right] + \frac{1}{2}\right\}\kappa & \text{if}\, \sigma \leq \sigma_c \\ \sqrt{6(\sigma^2+\sigma+1)}\frac{\kappa}{6} & \text{if}\, \sigma \geq \sigma_c, \end{cases} \tag{6}$$

Results were compared to ray-tracing and they show good accordance. Two different LE configurations were chosen: a) $r = 524$mm, $a = 0.3$mm, $t = 0.1$mm, $N = 100$; b) $r = 880$mm, $a = 1.7$mm, $t = 0.3$mm, $N = 120$. For these configurations, effective collecting length was calculated for various ζ. Titanium at unpolarized x-rays of 2keV energy was chosen as a material. Comparison of results of ray-tracing simulation and the analytical method are compared on Fig. 1.

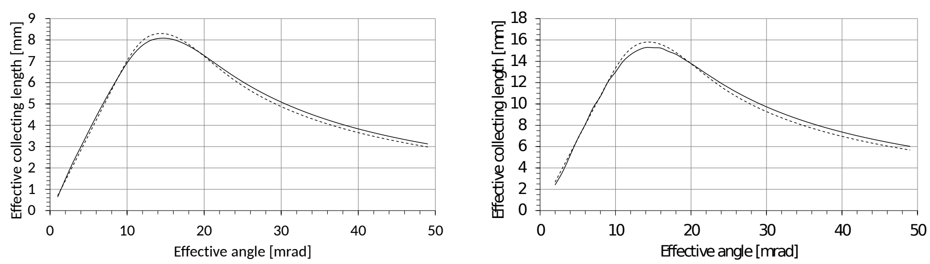


Figure 1: Comparison of results of ray-tracing simulations (solid line) and presented analytical method (dashed line).

Results were presented at AXRO conference, Prague, December 4-7, 2017 and submitted to impacted journal Contributions of the Astronomical Observatory Skalnaté Pleso.

3.2 Study of impact of manufacturing errors to performances of Schmidt lobster eye

Schmidt lobster eye (SLE) stack is composed of flat rectangular mirrors. The mirrors are held in their positions by a mechanical supporting structure. There are two types of manufacturing errors: 1) Mirrors are never ideally flat. 2) The supporting structure is never manufactured with ideal precision that causes deviations of mirror positions. Both these aspects and their impact to spatial resolution of SLE were studied.

Measurements of mirror specimens shown that mirrors have significantly different flatness in different axes (Fig. 2,Fig. 3). In one axis, the measured deformation seems that it is caused only by gravity and the method of holding. In the second axis, wave deformation of length approx. 1 μm has been measured.

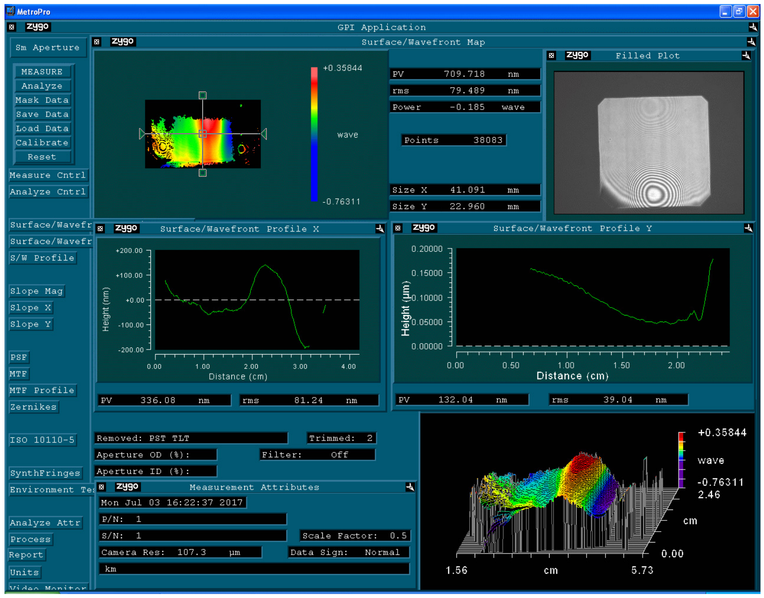


Figure 2: Typical mirror profiles. In the left-hand middle pannel, there is the lateral profile. In the right-hand middle pannel, there is the optical axis profile.

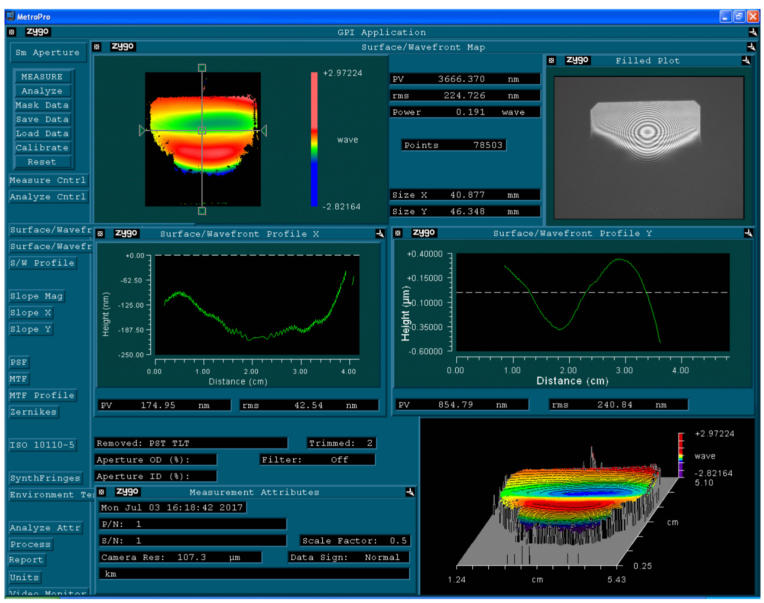


Figure 3: Typical mirror profiles. In the left-hand middle pannel, there is the optical axis profile. In the right-hand middle pannel, there is the lateral profile.

First, the simulation was performed with assumption that everything is ideally assembled. The resulting image is shown in Fig. 4. FWHM equals 0.4 mm. For simulations, parameters of prototype lobster eye LTW-51 were used. The prototype is one-dimensional, its focal length equals 51 cm. It is composed of 30 glass substrate mirrors of dimensions

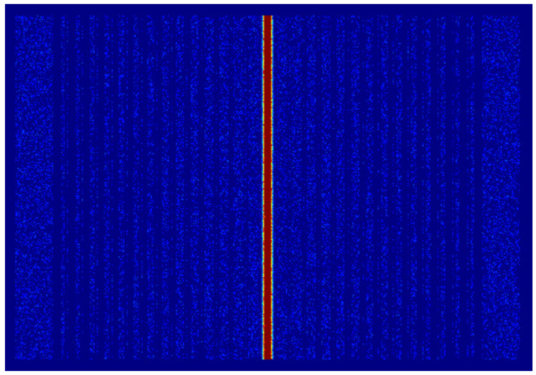


Figure 4: Results of simulation for ideally assembled SLE with ideally flat mirrors

37×37 cm, glass thickness equals 0.28 mm. Mirrors pitch between centres of surfaces is 0.995 mm. Mirrors are coated by gold with microroughness 1 μm. The prototype LTW-51 was assembled with inner mirror deformations placed laterally. The prototype was tested at energy 1490eV and the same value was used in ray-tracing simulations in QSOFT package.

During simulations of mirror inner deformation, it was supposed that all deformations are placed in one axis. Deformations were modelled as sine wave $\delta = A \sin(x/\lambda - \phi)$. Gaussian distribution of mean value 1μ m and $\sigma = 250$ nm was used for random generation of amplitude A. For period, Gaussian distribution of mean value 40 mm and $\sigma = 10$ mm was used. For generating of phase ω, uniform distribution on interval $[0, 2\pi)$ was used. These parameters were individually generated for each mirror. QSOFT software package was enhanced to simulate sine wave deformations.

Simulations of mirror inner deformation proved the following: If deformations are placed laterally, no apparent effect was observed and FWHM value stood at 0.4 mm. However, if deformations are placed in optical axis, image is blurred and FWHM increases to 1.1 mm, see Fig. 5.

Deviations of the supporting structure were simulated as random position errors in corners of mirrors. Gaussian distribution of various values of σ was used to generate these values. Between corners, linear interpolation was used to simulate the mirror profile.

Results for various values of σ are shown in Fig. 6

Experimentally acquired image (Fig. 7) by prototype SLE LTW-51 was obtained using the 35 m long X-ray beam-line of the XACT facility of INAF-OAPA in Palermo, Italy. Its FWHM reaches 0.7 mm. This value of FWHM was obtained in simulations supposing 5μm mirror positioning error. The small skew is caused by position shift between opposite parts of supporting structure and it was estimated as 10μm. This skew was simulated as systematic shift of mirror corner points. The above values were used for the final

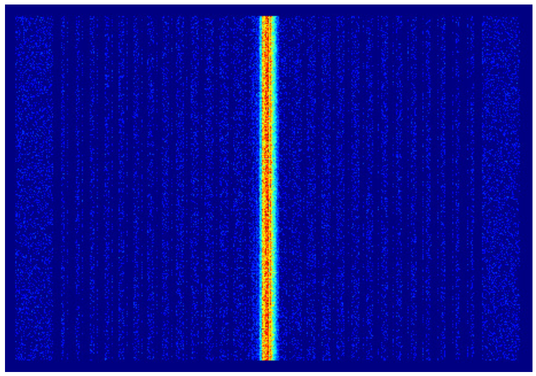
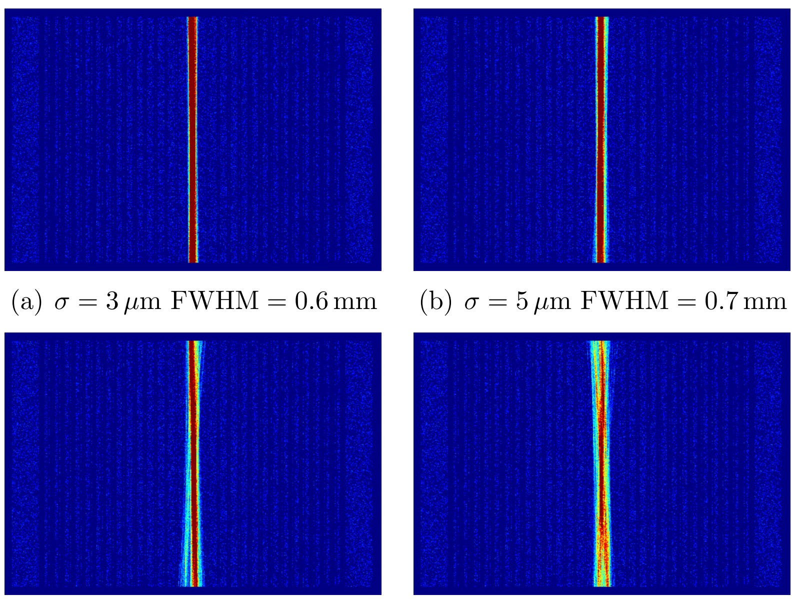


Figure 5: Results of simulation for ideally assembled SLE with mirrors including deformations in the optical axis



(a) $\sigma = 3\,\mu$m FWHM = 0.6 mm

(b) $\sigma = 5\,\mu$m FWHM = 0.7 mm

(c) $\sigma = 7\,\mu$m FWHM = 0.8 mm

(d) $\sigma = 10\,\mu$m FWHM = 1.2 mm

Figure 6: Simulation of deviations of supporting structure

simulation with the skew and supporting structure inaccuracy included. The resulting image is shown in Fig. 8

Results were presented at AXRO conference, Prague, December 4-7, 2017 and submitted to impacted journal Contributions of the Astronomical Observatory Skalnaté Pleso.

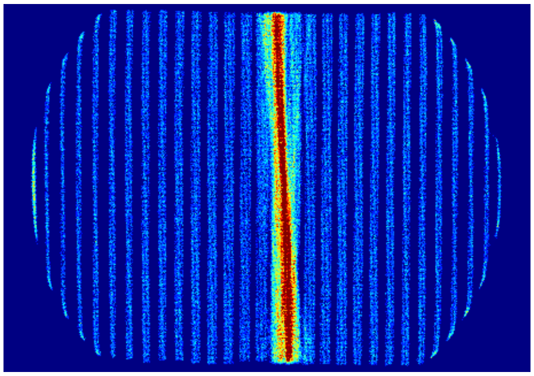


Figure 7: Experimental image

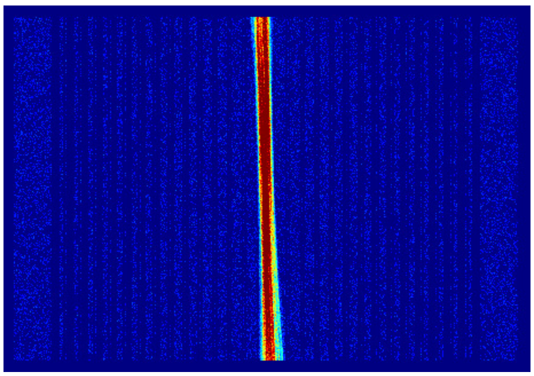


Figure 8: Simulation including skew error and mirror 10μ mpositioning error $\sigma = 5\mu$ m

3.3 Lobster eye Schmidt geometry using SPO

3.4 Lobster eye square pore geometry MPO