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ORIGINAL ARTICLE

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Optimization of mirror spacing or pore width of lobster eye optics

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Equations for the analytical calculation of gain and effective collecting area (length) of lobster eye systems are presented. The effect of mirror spacing (pore width) is analyzed, and equation for its optimal value with respect to effective collecting area (length) is found. The equations are applicable for Schmidt and Angel lobster eye designs.

KEYWORDS
instrumentation: Miscellaneous – telescopes

1 | INTRODUCTION

The lobster eye (LE) optics has been used and is proposed to be used in many astronomical instruments (e.g., Baca et al. 2016; Collier et al. 2015; Fraser et al. 2002, 2010; Gorenstein 2011; Owens et al. 2001; Petre et al. 2015; Tichý et al. 2015a). It is also being used in other applications, e.g., neutron imaging (Šaroun & Kulda 2006).

LE X-ray optics exists in two basic concepts: Schmidt (Schmidt 1975) and Angel (Angel 1979). Schmidt LE (Schmidt 1975) can be one dimensional or two dimensional. The basic one-dimensional Schmidt LE (SLE) stack is schematically shown in Figure 1. In a real case, the grazing angles are much smaller and the mirrors are closer together. The system is composed of flat rectangular mirrors. These mirrors form a uniform pattern around of a virtual cylinder of center C and radius r. The point F represents the focus of the system. The focal length of the optics is $f = r/2$ if it is composed of mirrors of negligible thickness. A two-dimensional Schmidt system is composed of two stacks of different radii (r_1, r_2) perpendicularly arranged.

The Angel LE (ALE) optics (Angel 1979) is composed of spherically arranged square pores. Approximately, the ALE can be viewed as a special case of the Schmidt system where both stacks lie in the same position and they have the same radii ($r = r_1 = r_2$). In this case, two stacks of mirrors form square pores. Although the spherical arrangement of the ALE is different from the cylindrical arrangement of the SLE, the difference is small. This is because the active part of the system is limited by the reflectivity function and the reflectivity falls to zero at small angles of a few degrees or less than a degree. In such a small region, the approximation is admissible.

For numerical simulations of LE optics, the general ray-tracing approach is possible (see, e.g., Spencer & Murty 1962) or its simplified versions (Šaroun & Kulda 2006; Tichý et al. 2016).

Analytic models are approximate but they express the result (e.g., effective collecting area, gain, etc.) as a mathematical function of the parameters. They are useful for approximate but fast estimation of the performance. Analytic models are also useful for finding the initial point for subsequent optimization by ray-tracing simulations. In addition, analytical models provide information on how the result depends on the initial parameters. Some analytic equations for the lobster system have already been presented (Angel 1979; Inneman 2001; Schmidt 1975) but they do not include all the parameters; e.g., zero mirror thickness is assumed. Su et al. (2017) presented an analysis of the focusing efficiency as a function of the X-ray wavelength, but the final results were based on ray-tracing simulations of specific examples. Chapman et al. (1991) presented a comprehensive analysis of the efficiency, but the source was supposed to be at a finite distance and the source lay on the concave side of the LE optics. This configuration corresponds to, e.g., microscopes; however, the configuration of telescopes is different. For application in telescopes, the source must be at an infinite distance on the convex side of the LE optics.

Abbreviations: ALE, Angel lobster eye; LE, lobster eye; SLE, Schmidt lobster eye.

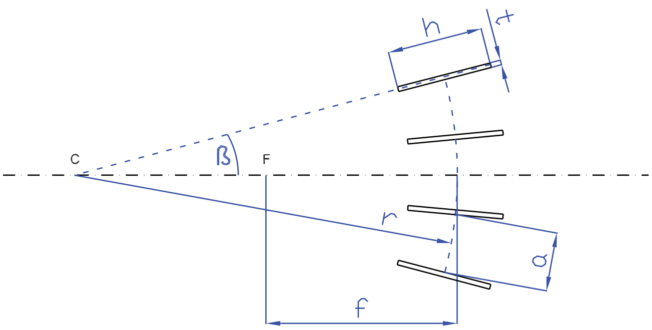

FIGURE 1 Lobster eye geometrical parameters

This paper follows the previous work of some of the authors (Tichý et al. 2015b, 2018). The effective collecting area (length) and the gain are expressed as functions of the LE parameters. Equations for the optimal parameters that maximize the effective collecting area (length) are presented.

2 | GENERAL EQUATION

As in several papers (Chapman et al. 1991; Schmidt 1975; Su et al. 2017; Tichý et al. 2015b, 2016), the problem of a single LE stack is analyzed in the cross-sectional plane. The geometry of the stack is described by the following parameters (Figure 1): radius of the system r; middle mirror spacing from surface to surface (pore width) a; mirror thickness (pore wall width) t, mirror (pore) depth h, and the number of mirrors N.

The parameter $\zeta \approx \arctan(a/h) \approx a/h$ is called the effective angle (Inneman 2001; Schmidt 1975; Tichý et al. 2015b). In this paper, ζ will be used as an independent parameter instead of h. The variable β represents the angular position of a mirror. Parallel incoming rays are assumed, and therefore $\beta = \theta$, where θ is the grazing angle of reflection.

The general equation for the effective length, including the mirror reflectivity and all geometrical parameters, has been derived as (Tichý et al. 2015b, 2018)

$$L \approx \frac{2r}{a+t}\left[\int_0^{\zeta} h\theta\mathcal{R}(\theta)d\theta + \int_{\zeta}^{2\zeta}(2a - h\theta)\mathcal{R}(\theta)d\theta\right]. \quad (1)$$

Here, it was assumed that the mirrors are placed at least in all angular positions between $\pm 2\zeta$, i.e., their number fulfills the condition $N > 2\frac{a}{a+t}\frac{r}{h}$. $\mathcal{R}(\theta)$ is the reflectivity function.

Because $\int_p^q \theta\mathcal{R}(\theta)d\theta = \left[\theta\int\mathcal{R}(\theta)d\theta\right]_{\theta=p}^{\theta=q} - \int_p^q\int\mathcal{R}(\theta)d\theta d\theta$, Equation 1 can be modified to the following form (Tichý et al. 2018):

$$L(r, a, t, \zeta) = 2r\frac{a}{a+t}K(\zeta), \quad (2)$$

where

$$K(\zeta) := \frac{\widetilde{\mathcal{R}}(2\zeta) - 2\widetilde{\mathcal{R}}(\zeta) + \widetilde{\mathcal{R}}(0)}{\zeta} \quad (3)$$

and $\widetilde{\mathcal{R}}(\theta) := \int\int\mathcal{R}(\theta)d\theta d\theta$ is an arbitrary second antiderivative of $\mathcal{R}$. Equations 2 and 3 allow us to calculate the effective collecting length of an LE of given geometrical parameters with a given model of reflectivity. These equations represent a single stack that is inspected in a cross-sectional plane, and therefore we refer to the collecting length here rather than the collecting area. Its transformation to the effective collecting area is simple. For the one-dimensional system, L is multiplied by the mirror width to get the effective area. The effective collecting area of the Angel system equals L^2, whereas the effective collecting area of a Schmidt lobster system composed of two stacks of effective areas $L_1(r_1, a_1, t_1, \zeta_1)$ and $L_2(r_2, a_2, t_2, \zeta_2)$ equals L_1L_2.

Equations 2 and 3 clearly show the form of the dependence on some parameters. First, it is seen that the effective collecting length is proportional to the radius of the system. Second, the effective collecting length is proportional to $a/(a+t)$. This term can be interpreted as the ratio between the total size of the input aperture and the part that is shaded by the sides of the mirrors (Inneman 2001; Tichý et al. 2015b). For a system composed of ideal mirrors of zero thickness, this fraction would be equal to 1.

The function K depends on the effective angle, i.e., the ratio between the mirrors' depth and spacing, and the reflectivity function only. The calculation and maximization of K for various reflectivity models was presented by Tichý et al. (2018). There are equations for the optimal value of ζ. For example, for a model is called the smoothed step, defined as

$$\mathcal{R}(\theta) = \begin{cases} Q & \forall 0 \le \theta \le \rho \\ Q\frac{\kappa-\theta}{\kappa-\rho} & \forall \rho \le \theta \le \kappa \\ 0 & \forall \theta \ge \kappa \end{cases}. \quad (4)$$

here, θ is the grazing angle, and $0 < \rho < \kappa$ and $0 < Q \le 1$ are constants. The optimal value of ζ is given by

$$\zeta_{\text{optimal}} = \begin{cases} \left\{\cos\left[\frac{1}{3}\arccos(\sigma^3) + \frac{\pi}{3}\right] + \frac{1}{2}\right\}\kappa & \text{if } \sigma \le \sigma_c \\ \sqrt{6(\sigma^2 + \sigma + 1)}\frac{\kappa}{6} & \text{if } \sigma \ge \sigma_c, \end{cases} \quad (5)$$

where $\rho = \kappa\sigma$; $0 < \sigma < 1$, and $\sigma_c = \frac{1+\sqrt{21}}{10} = 0.558$.

3 | OPTIMAL VALUE OF MIRROR SPACING

The nirror thickness t can be considered to be governed by other requirements, e.g., the mechanical integrity. By Equation 2, it should be as small as possible to achieve the largest collecting length.

There is one free parameter a (mirror spacing/pore width) left. Analysis of the effect of this parameter on the performance of an LE system is the subject of this paper.

Let $R = (r + h/2)$ be the total distance between the focus and the front aperture, i.e., the total length of the telescope. This is a value that can be also given by design considerations.

After substitution into Equation 2, we obtain

$$L(r, a, t, \zeta) = \frac{2R\zeta a - a^2}{\zeta(a+t)}K(\zeta). \quad (6)$$

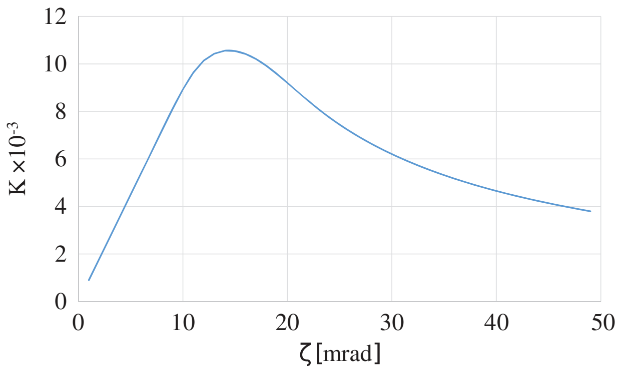

FIGURE 2 Graph of the function K versus the effective angle ζ

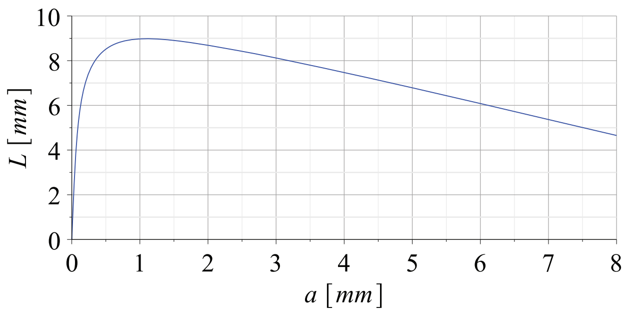

FIGURE 3 Graph of the effective collecting length L versus the mirror spacing (pore width) a

The maximum of the function 6 is easy to find by common methods of mathematical analysis. There exists just one maximum of function 6, which is

$$a_{\text{optimal}} = -t + \sqrt{2Rt\zeta + t^2}. \tag{7}$$

4 | EXAMPLE

A system of total length $R = 500$ mm composed of a mirror of thickness $t = 0.1$ mm is assumed. Mirrors are coated by titanium, and the photon energy is equal to 2 keV. The smoothed step model parameters are $Q = 0.9$ and $\rho = 17$ mrad, and $\kappa = 23.5$ mrad is chosen as the relevant approximation of reflectivity. The graph of K versus ζ is shown in Figure 2. By Equation 5, the optimal value of $\zeta = 14.4$ mrad corresponds $K = 0.0106$.

The value $\zeta = 14.4$ mrad is consequently used to find the optimal value of a. The graph of L versus a given by Equations 2 and 3 is shown in Figure 3.

By Equation 7, the optimal value of $a = 1.10$ mm. In this optimal case, the effective collecting length calculated by Equations 2 and 3 is $L = 8.97$ mm.

5 | GAIN

Because incoming rays are focused to a spot of size a (Schmidt 1975), it is easy to express the gain G as

$$G = \frac{L}{a} = \frac{2R\zeta - a}{\zeta(a+t)} K(\zeta). \tag{8}$$

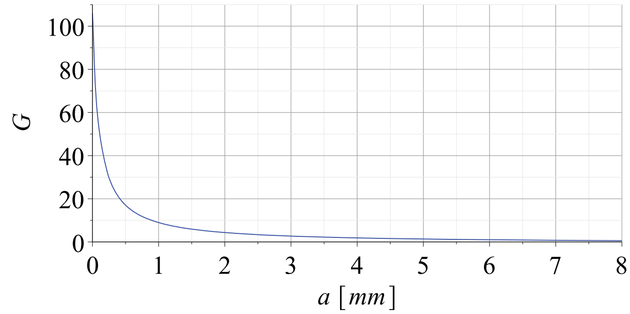

FIGURE 4 Graph of the gain G versus the mirror spacing (pore width) a

However, the function 8 is monotonically decreasing and there is no optimal value of a with respect to gain. This is because the spot size is equal to a only if $a \gg t$. Therefore, the equation is not valid for small values of a. Searching for a general formula for gain is the subject of further research.

The graph of gain G versus a for the same example as in the previous section is shown in Figure 4.

6 | CONCLUSIONS

It was found that for a given total length of telescope, mirror thickness, and mirror reflectivity, there exist optimal values of all remaining geometrical parameters of the LE. The aim of this paper was to find an optimal value of mirror spacing of SLE or the pore width of ALE. The equation for gain was derived, but it is valid only if the mirror spacing (pore width) is much larger than mirror thickness (pore-wall width), and therefore the equation does not allow us to find the optimal value of mirror spacing (pore witdth) with respect to gain.

The equations presented here were derived for a one-dimensional stack of cylindrically arranged flat mirrors, i.e., for a one-dimensional SLE. They can easily be applied to a two-dimensional SLE composed of two such stacks. Calculations can be made for each stack independently. The resulting gain or effective collecting area is calculated as the product of the gain or effective collecting lengths of both stacks. In the case of ALE, the gain or the effective collecting area is approximately equal to the square of the gain or the effective collecting length of a 1-D Schmidt system.

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Conflicts of interest

The authors declare no potential conflicts of interest.

Financial disclosure

None reported.

Author contributions

V.T. carried out the analysis. R.W. managed the paper preparation and language corrections. All authors discussed the results and contributed to the final manuscript.

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